Snakes related strongly*-graphs

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Abstact *—A graph with n vertices is said to be strongly*-graph if its vertices can be assigned the values $\{1, 2, \ldots, n\}$ in such a way that when an edge whose end vertices are labeled i and j, is labeled with the value $i+j+ij$ such that all edges have distinct labels. Here we derive some snakes related strongly*-graphs.*
Keyword* - *Strongly*-labeling, Strongly*-graph.
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I. INTRODUCTION

Here graph G is considered as a simple, finite, undirected graph.

Definition I.1. *[1] A graph G with n vertices is said to be strongly*-graph if there is a bijection $f : V(G) \longrightarrow \{1, 2 \ldots, n\}$ such that the induced edge function $f^* : E(G) \longrightarrow \mathbb{N}$ defined as $f^*(e = uv) = f(u) + f(v) + f(u) \cdot f(v)$ is injective. Here f is called strongly*-labeling of graph G.*

II. MAIN RESULTS

Definition II.1. *[2] A triangular snake T_n is obtained from a path P_n by replacing each edge of the P_n by a cycle C_3.*

Definition II.2. *[4] An alternate triangular snake $A(T_n)$ is obtained from a path P_n by replacing each alternate edge of P_n by a cycle C_3.*

Theorem II.1. *Alternate triangular snake $A(T_n)$ is strongly*-graph.*

Proof. Let $A(T_n)$ be the alternate triangular snake obtained from a path P_n with consecutive vertices $\{u_1, u_2, \ldots, u_n\}$ by joining u_i and u_{i+1} with new vertex v_j alternatively, $1 \leq i \leq n-1$, $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$.
To define vertex labeling function $f : V(A(T_n)) \rightarrow \{1, 2, \ldots, |V(A(T_n))|\}$ we consider the following cases.
Case 1: Triangle in $A(T_n)$ starts from u_1.
Note that in this case $|V(A(T_n))| = \lfloor \frac{n}{2} \rfloor + n$ and $|E(A(T_n))| = \begin{cases} 2n-1, & \text{if } n \text{ is even.} \\ 2(n-1), & \text{if } n \text{ is odd.} \end{cases}$
$f(u_i) = \lfloor \frac{n}{2} \rfloor + n$, $1 \leq i \leq n$.
$f(v_j) = 3j - 1$, $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$.
Strongly*-labeling of $A(T_n)$ in this case is shown in *Figure 1*.

Case 2: Triangle in $A(T_n)$ starts from u_2.
Subcase 1: n is odd.
Defined as case-1.
Subcase 2: n is even.
Note that $|V(A(T_n))| = \frac{3n}{2} - 1$ and $|E(A(T_n))| = 2n - 3$
$f(u_i) = \frac{3i}{2} - 1$, $1 \leq i \leq n$.
$f(v_j) = 3j$, $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$.
Strongly*-labeling of $A(T_n)$ in this case is shown in *Figure 2*.

Here the vertex labels produced by labeling f in $A(T_n)$ are strictly increasing.

Hence obviously edge labels produced by $f^*(uv) = f(u) + f(v) + f(u) \cdot f(v)$ are increasing for all pair of vertices u and v in $A(T_n)$. Hence the edge labels produced are all distinct. So, the labeling pattern defined above satisfies the conditions of strongly *-labeling. i.e. $A(T_n)$ is strongly*-graph. □

Definition II.3. *[3] A double triangular snake DT_n consists of two triangular snakes that have a common path.*

Theorem II.2. *Double triangular snake DT_n is strongly*-graph.*

Proof. Let $\{u_1, u_2, \ldots, u_n\}$ be successive vertices of P_n in DT_n, where u_i is adjacent to u_{i+1}, $1 \leq i \leq n-1$. Join u_i and u_{i+1} with two new vertices v_i and w_i, $1 \leq i \leq n-1$.
Here, $|V(DT_n)| = 3n - 2$ and $|E(DT_n)| = 5(n-1)$.
We define vertex labeling function $f : V(DT_n) \rightarrow \{1, 2, \ldots, |V(DT_n)|\}$ as follows.
$f(u_i) = 3i - 2$, $1 \leq i \leq n$.

$$\left.\begin{array}{l} f(v_i) = 3i - 1 \\ f(w_i) = 3i \end{array}\right\} 1 \leq i \leq n-1.$$

Strongly*-labeling of DT_n is shown in *Figure 3*.

Since the vertex labels are strictly increasing, for any pair of adjacent vertices (u_i, v_j) and (u_s, u_t), we have $(f(u_i) + f(v_j) + f(u_i)f(v_j)) \neq (f(u_s) + f(u_t) + f(u_s)f(u_t)) \Rightarrow f^*(u_iv_j) \neq f^*(u_sv_t)$. Hence above defined labeling pattern satisfies the conditions of strongly*-labeling. i.e. Double triangular snake is strongly*-graph. □

Definition II.4. *[4] An alternate double triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have common path.*

Theorem II.3. *Alternate double triangular snake $DA(T_n)$ is strongly*-graph.*

Proof. Let $DA(T_n)$ be the alternate double triangular snake obtained from a path P_n with consecutive vertices $\{u_1, u_2, \ldots, u_n\}$ by joining u_i and u_{i+1} with two new vertices v_j and w_j alternatively, $1 \leq i \leq n-1$ and $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$.
In this case we define vertex labeling function $f : V(DA(T_n)) \rightarrow \{1, 2, \ldots, |V(DA(T_n))|\}$ as follows.
Case 1: Triangle in $DA(T_n)$ starts from u_1.
Note that $|V(DA(T_n))| = \begin{cases} 2n, & \text{if } n \text{ is even.} \\ 2n-1, & \text{if } n \text{ is odd.} \end{cases}$ and $|E(DA(T_n))| = \begin{cases} 3n-1, & \text{if } n \text{ is even.} \\ 3n-3, & \text{if } n \text{ is odd.} \end{cases}$

$f(u_i) = 2i - 1$, $1 \leq i \leq n$.

$$\left.\begin{array}{l} f(v_j) = 4j - 2, \\ f(w_j) = 4j, \end{array}\right\} 1 \leq j \leq \lfloor \frac{n}{2} \rfloor.$$

In this case strongly*-labeling of $DA(T_n)$ is shown in *Figure 4*.
Case 2: Triangle in $DA(T_n)$ starts from u_2.
Subcase 1: n is odd.
Defined as case-1.
Subcase 2: n is even.
Note that $|V(DA(T_n))| = 4n - 2$ and $|E(DA(T_n))| = 3n - 5$.
$f(u_1) = 1$.
$f(u_i) = 2(i-1)$, $2 \leq i \leq n$.

$$\left.\begin{array}{l} f(v_j) = 4j - 1, \\ f(w_j) = 4j + 1, \end{array}\right\} 1 \leq j \leq \lfloor \frac{n}{2} \rfloor.$$

In this case strongly*-labeling of $DA(T_n)$ is shown in *Figure 5*.
It can be easily observed that if vertex labels are in increasing order so, the produced edge labels are also increasing. So, they produced different edge labeling.

Hence above defined labeling pattern satisfies the conditions of strongly *-labeling. i.e. $DA(T_n)$ is strongly*-graph.

□

Definition II.5. *[3] A quadrilateral snake Q_n is obtained from a path P_n by replacing each edge of P_n by a cycle C_4.*

Theorem II.4. *Quadrilateral snake Q_n is strongly*-graph.*

Proof. Let P_n be the path of $\{u_1, u_2, \ldots, u_n\}$ to construct Q_n, join u_i and u_{i+1}(alternatively) to two new vertices v_i and w_i by the edges u_iv_i, $u_{i+1}w_i$ and v_iw_i, $1 \leq i \leq n-1$.
Here $|V(Q_n)| = 3n - 2$ and $|E(Q_n)| = 4(n-1)$.
We define vertex labeling function $f : V(Q_n) \rightarrow \{1, 2, \ldots, |V(Q_n)|\}$ as follows.
$f(u_i) = 3i - 2$, $1 \leq i \leq n$.

$$\left.\begin{array}{l} f(v_i) = 3i - 1, \\ f(w_i) = 3i, \end{array}\right\} 1 \leq i \leq n-1.$$

Strongly*-labeling of Q_n is shown in *Figure 6*.

Here every edge of P_n is an edge of cycle C_4 having the vertices u_i, v_i, w_i and u_{i+1} which are labeled in strictly increasing. So, it is obvious that for any pair of adjacent vertices produce different edge labeling for defined function. So, they satisfies the conditions of strongly *-graph. i.e. Q_n is strongly*-graph.

□

Definition II.6. *[4] An alternate quadrilateral snake $A(Q_n)$ is obtained from a path P_n, each alternate edge of P_n is replaced by a cycle C_4.*

Theorem II.5. *Alternate quadrilateral snake $A(Q_n)$ is strongly*-graph.*

Proof. Let $A(Q_n)$ be the alternate quadrilateral snake obtained from a path P_n with consecutive vertices $\{u_1, u_2, \ldots, u_n\}$ by joining u_i and u_{i+1} (alternatively) with new vertices v_j and w_j, $1 \leq i \leq n-1$ and $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$.
To define vertex labeling function $f : V(A(Q_n)) \rightarrow \{1, 2, \ldots, |V(A(Q_n))|\}$ we consider the following cases.
Case 1: Quadrilateral in $A(Q_n)$ starts from u_1.
Note that $|V(A(Q_n))| = \begin{cases} 2n, & \text{if } n \text{ is even.} \\ 2n-1, & \text{if } n \text{ is odd.} \end{cases}$ and $|E(A(Q_n))| = \begin{cases} \frac{5n}{2} - 1, & \text{if } n \text{ is even.} \\ 5(n-1), & \text{if } n \text{ is odd.} \end{cases}$

$$f(u_i) = \begin{cases} 2i, & \text{if } i \text{ is even.} \\ 2i-1, & \text{if } i \text{ is odd.} \end{cases}$$

$$\left.\begin{array}{l} f(v_j) = 4j - 2, \\ f(w_j) = 4j - 1, \end{array}\right\} 1 \leq j \leq \lfloor \frac{n}{2} \rfloor.$$

In this case strongly*-labeling of $A(Q_n)$ is shown in *Figure 7*.

Case 2: Quadrilateral in $A(Q_n)$ starts from u_2.
Subcase 1: n is odd.
Defined as case-1.
Subcase 2: n is even.
Note that $|V(A(Q_n))| = 2(n-1)$ and $|E(A(Q_n))| = \frac{5n}{2} - 4$.
$f(u_i) = 2(i-1)$, $1 \leq i \leq n$.

$$\left.\begin{array}{l} f(v_j) = 4j - 1, \\ f(w_j) = 4j, \end{array}\right\} 1 \leq j \leq \lfloor \frac{n}{2} \rfloor.$$

In this case strongly*-labeling of $A(Q_n)$ is shown in *Figure 8*.

In above cases every alternate edge of path is related to cycle C_4 and defined labeling function f for $A(Q_n)$ are strictly increasing. So, the induced edge labels by $f^*(u_iv_j) = f(u_i) + f(v_j) + f(u_i) \cdot f(v_j)$, $f^*(w_jv_j) = f(w_j) + f(v_j) + f(w_j) \cdot f(v_j)$, $f^*(u_iw_j) = f(u_i) + f(w_j) + f(u_i) \cdot f(w_j)$ and $f^*(u_iu_{i+1}) = f(u_i) + f(u_{i+1}) + f(u_i) \cdot$

$f(u_{i+1})$ are increasing and distinct for all pair of adjacent vertices (u_i, u_{i+1}), (u_i, v_j), (w_j, v_j) and (w_j, u_{i+1}) in $A(Q_n)$. Hence above defined labeling pattern satisfies the conditions of strongly*-labeling. i.e. $A(Q_n)$ is strongly*-graph □

Definition II.7. *[3] A double quadrilateral snake DQ_n is consist of two quadrilateral snakes that have common path.*

Theorem II.6. *Double quadrilateral snake DQ_n is strongly*-graph.*

Proof. Let P_n be the path of $\{u_1, u_2, \ldots, u_n\}$ to construct DQ_n, join u_i and u_{i+1}(alternatively) to four new vertices v_i, w_i, v'_i and w'_i by the edges u_iv_i, $u_{i+1}w_i$, v_iw_i, $u_iv'_i$, $u_{i+1}w'_i$ and $v'_iw'_i$, $1 \le i \le n-1$.
Here $|V(DQ_n)| = 5n - 4$ and $|E(DQ_n)| = 7(n-1)$.
We define vertex labeling function $f : V(DQ_n) \to \{1, 2, \ldots, |V(DQ_n)|\}$ as follows.

$f(u_1) = 1.$
$f(u_i) = 5i - 6,\ 2 \le i \le n.$

$$\left.\begin{aligned} f(v_i) &= 5i-3,\\ f(w_i) &= 5i-2,\\ f(v'_i) &= 5i+1,\\ f(w'_i) &= 5i, \end{aligned}\right\} 1 \le i \le n-1.$$

Strongly*-labeling of DQ_n is shown in *Figure 9*.

Here every edge of P_n is an edge of cycle C_6 with one chord having the vertices u_i, v_i, w_i, u_{i+1}, w'_i and v'_i which are labeled in strictly increasing. So, it is obvious that for any pair of adjacent vertices (u_i, v_i), (v_i, w_i), (u_i, u_{i+1}), (w'_i, v'_i), (w'_i, u_{i+1}) and (u_i, v'_i) produce different edge labeling for defined function. So, they satisfies the conditions of strongly *-graph. i.e. DQ_n is strongly*-graph.

□

Definition II.8. *[4] An alternate double quadrilateral snake $A(DQ_n)$ is consist of two alternate double quadrilateral snakes that have common path.*

Theorem II.7. *Alternate double quadrilateral snake $DA(Q_n)$ is strongly*-graph.*

Proof. Let $DA(Q_n)$ be the alternate double quadrilateral snake obtained from a path P_n with consecutive vertices $\{u_1, u_2, \ldots, u_n\}$ by joining u_i and u_{i+1} (alternatively) with four new vertices v_j, w_j, v'_j and w'_j by the edges u_iv_j, $u_{i+1}w_j$, v_jw_j, $u_iv'_j$, $u_{i+1}w'_j$ and $v'_jw'_j$, $1 \le i \le n$ and $1 \le j \le \lfloor \frac{n}{2} \rfloor$.
To define vertex labeling function $f : V(DA(Q_n)) \to \{1, 2, \ldots, |V(DA(Q_n))|\}$ we consider the following cases.
Case 1: Quadrilateral in $DA(Q_n)$ starts from u_1.
Note that $|V(DA(Q_n))| = \begin{cases} 3n, & \text{if } n \text{ is even.} \\ 3n-2, & \text{if } n \text{ is odd.} \end{cases}$ and $|E(DA(Q_n))| = \begin{cases} 4n-5, & \text{if } n \text{ is even.} \\ 4(n-1), & \text{if } n \text{ is odd.} \end{cases}$

$f(u_i) = 3i - 2,\ 1 \le i \le n.$

$$\left.\begin{aligned} f(v_j) &= 6j-4,\\ f(w_j) &= 6j-3,\\ f(v'_j) &= 6j,\\ f(w'_j) &= 6j-1, \end{aligned}\right\} 1 \le j \le \lfloor \frac{n}{2} \rfloor.$$

In this case strongly*-labeling of $DA(Q_n)$ is shown in *Figure 10*.

Case 2: Quadrilateral in $DA(Q_n)$ starts from u_2.
Subcase 1: n is odd.
Defined as case-1.
Subcase 2: n is even.
Note that $|V(DA(Q_n))| = 3n - 4$ and $|E(DA(Q_n))| = 4n - 7$.
$f(u_1) = 1.$
$f(u_i) = 3i - 4,\ 2 \le i \le n.$

$$\left.\begin{aligned} f(v_j) &= 6j-3,\\ f(w_j) &= 6j-2,\\ f(v'_j) &= 6j+1,\\ f(w'_j) &= 6j, \end{aligned}\right\} 1 \le j \le \lfloor \frac{n}{2} \rfloor.$$

In this case strongly*-labeling of $DA(Q_n)$ is shown in *Figure 11*.

It can be easily observed that if vertex labels are strictly increasing then the produced edge labels are also increasing. That means the edge labels produced are all distinct. Hence above defined labeling pattern satisfies the conditions of strongly *-labeling. i.e. $DA(Q_n)$ is strongly*-graph. □

III. FIGURES

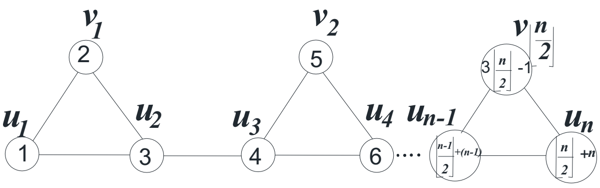

Fig. 1

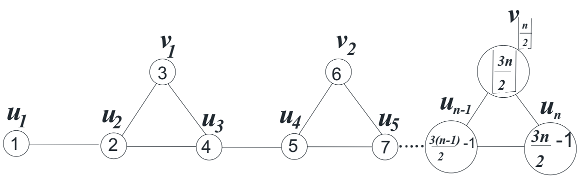

Fig. 2

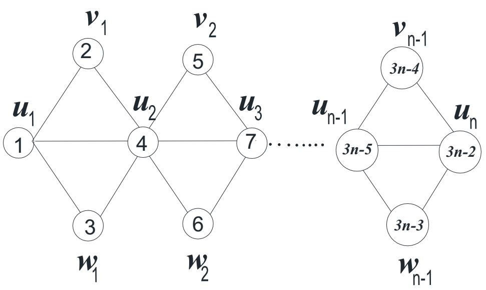

Fig. 3

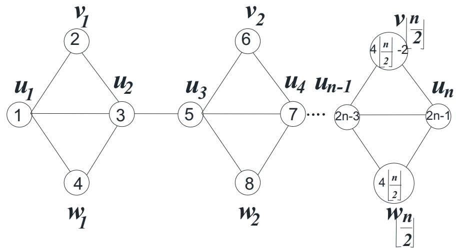

Fig. 4

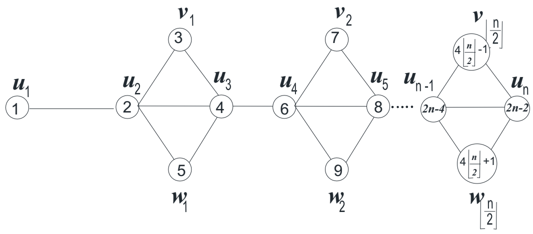

Fig. 5

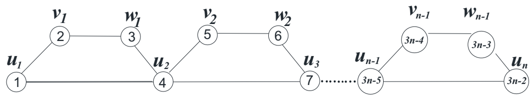

Fig. 6

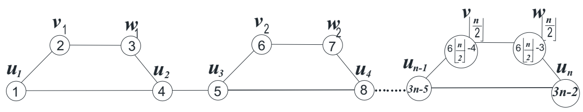

Fig. 7

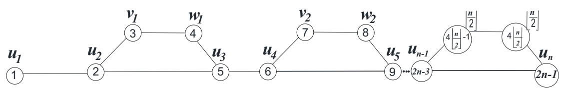

Fig. 8

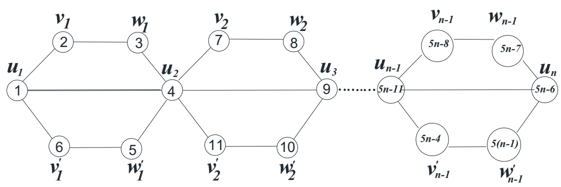

Fig. 9

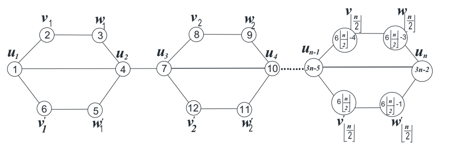

Fig. 10

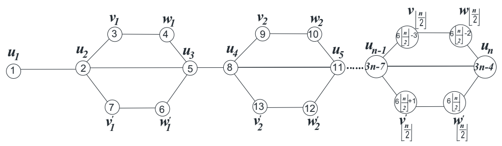

Fig. 11

IV. CONCLUSION

It has been proved that all cycles and paths admit labeling specific manner produces specific snakes-type graphs. It is intrusting to see whether these combination admit strong * labeling or not.

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